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B-Spline Surface Reconstruction by Control Point Optimization using Genetic Algorithm

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***Abstract*—The conventional engineering technologies such as computer aided geometric design and computer aided manufacturing systems are used to design and create physical objects from digital models. The reverse of this procedure i.e. transforming the real physical objects into a digital description is referred as reverse engineering. Surface reconstruction is the problem of recovering 3D shapes. It arises in a wide variety of applications such as CAD design, data visualization, virtual reality, medical imaging, computer animation, computer vision, computer graphics, scientific computing etc. In this paper, GA optimization approach is applied in order to reconstruct B-spline surface from a set of 3D data points. Surface reconstruction consists of data collection, data pre-processing, surface parameterization and surface fitting. In the present work, the 3D data points are obtained by cad modelling using MATLAB for a known object shapes. Data Pre-processing is used to arrange the data points in a proper sequence. Surface Parameterization is used for the determination of all relevant surface data such as knot vectors, control points. GA optimization approach is applied to optimize the location of control points i.e. proper placement of control points. GA is used because it constitutes a class of search algorithms especially suited to solving complex optimization problems. Genetic algorithm is a robust stochastic based search method and it maintains a population of potential solutions. The fitting surface is calculated by least squares through SVD (singular value decomposition) method. The SVD method is used because of its robustness and it provides good quality numerical answers. The proposed method yields good results and provide fine accuracy and flexibility with minimum computational effort.**

***Keywords*— Surface reconstruction, B-spline surface, Genetic algorithm, Surface parameterization, Surface fitting.**

I. INTRODUCTION

Three – dimensional (3D) digitization is a process to obtain a digital description from the real physical objects, which is also known as reverse engineering. There are various properties of a 3D object that may be recovered, such as its shape, its colour, and its material properties. The problem of recovering 3D shapes is called as surface reconstruction. It is widely used in CAD design, data visualization, virtual reality, medical imaging, computer animation, computer vision, computer graphics, scientific computing, aerospace, automobile, biomedical, and consumer product industries to facilitate product design, analysis and manufacturing from pre-existing products [3], [4], [11], [13].

The problem of surface reconstruction [9]-[17], [24], [25] can be roughly divided into two phases, first phase consists of data acquisition, it involves a set of 3D data points which contain the partial information about the unknown object surface. The data points can be obtained by 3D scanner such as coordinate measuring machine [5], laser scanner, photometric system etc. and they may be either organized or scattered. Second phase consists of post sensing data pre-processing where a shape model is reconstructed. That is the data points are used to create a surface model that approximates the unknown object surface.

A. Reported Work

Reference [4] used two different schemes, SOM (self-organizing map) neural network and PDEs (partial differential equations) for the surface parameterization, and GDA (gradient descent algorithm) and RSEC (random surface error correction algorithm) for fitting a 3D surface to get the reconstructed surface. This method requires data points to be projected onto the base surface and therefore, only examples of neither self intersecting nor high-genus surfaces are properly handled. Reference [25] combined NURBS with Constructive Solid Geometry in a hybrid evolutionary algorithm/genetic programming approach. This method is simple and less powerful since no parameterization of data points is computed. The method is limited to very simple examples in which a planar base surface can be used and reported errors for data points are quite large. Reference [24] applied simulated annealing for optimizing NURBS ship hull fitting. This work performed surface reconstruction from a set of cross-sections called NURBS skinning rather than from clouds of points. A set of data points are used to generate a set of cross-sections for surface fitting by making all those cross-sections compatible and joining them with one another. Then, surface optimization is accomplished using simulated annealing. Errors with this method are quite large, and this method is limited to very specific surfaces for which suitable cross-sections can be obtained.

Reference [23] applied multi-objective evolutionary algorithm (MOEA) approach to reconstruct a simple smooth surface with different sets of data points. This method is limited since it is assumed that sampled points are uniformly spaced in the (u, v) domain, so clouds of scattered data points cannot be reconstructed. Reference [12] used evolutionary algorithms to recover the shape of tessellated surfaces such as a sphere, a fractal surface. For a population size of 30–50 particles, a polygonal mesh is obtained. The method is fails, where a proper triangulation cannot be obtained. The surface is linearly approximated, points on the reconstructed surface different from data points exhibit very large errors. Reference [15] used an automatic surface reconstruction method. The method is composed of quadrangle frame generation, point and curve networks planning, and surface patches reconstruction. In the first phase, the original triangle mesh is reduced and converted into a quadrangle mesh. In the second phase, the boundary data of the surfaces are prepared. These include a network of serial points, frame curves and surface normal which are also expressed as curves. In the final phase, surface initialization, harmonization mapping and surface warping are presented to yield the desired surfaces. The main advantage of this method is that it can relax the pre-processing of a scanned triangle mesh, and hence, increase the efficiency and quality of the surface reconstruction. Reference [11] used genetic algorithm approach for tensor-product B-spline surface fitting. The fitting surface is calculated by least-squares through either singular value decomposition or lower upper modification method. This method yields good results even in presence of problematic features and fitting errors are small.

In this paper, the surface reconstruction problem is solved by using a genetic algorithm approach. The proposed method is briefly summarized in Fig.1. A set of 3D data points obtained by cad modelling using MATLAB for a known object shapes, are fitted with a B-spline surface of a certain order. In order to do so, genetic algorithm techniques are applied to determine control points. Then, the fitting surface is calculated by LSQ (least squares) through SVD (singular value decomposition). Reconstructed data points are then compared with the original ones. This pipeline is applied iteratively until a prescribed threshold error for data points is achieved.

II. PROPOSED METHOD

A. Data acquisition and pre-processing

In the present work, the 3D data points are obtained by cad modelling using MATLAB for a known object shapes. A cylindrical object is taken to obtain the data points. To create data points the known surface is divided into a small number of grid sizes and each coordinates of grid point are stored. These grid coordinates value are put in the known surface equation and finding the coordinate of that point for which function value is zero or less than a specified accuracy. The Coordinates that are satisfying this condition are lies on or near the known surface i.e. on the periphery of the circular face of cylinder. Then these points are stored as initial data points.

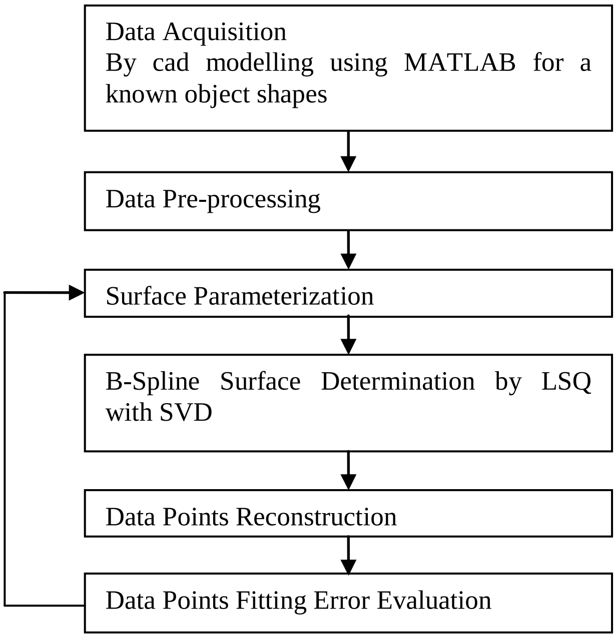

Fig. 1. Workflow of the Proposed Method

A known object shape cylinder having radius 5 and height 5 units is taken for data acquisition. The grid size for x×y×z axis is taken as 0.1×0.1×0.1. For 0.1 unit accuracy 3876 numbers of 3D data points are obtained. Data Pre-processing [9] is needed to separate the noise from the surface in case of noisy data points. It involves trimming of the data coming from near the edges of the surfaces, as the noise in such regions has quite different characteristics as that from the interior points. Then a smooth surface is fitted around each measured surface point. For plotting a spline curve the points are needed in a sequence from which the curve has passed. For doing this the data points have sorted in such a sequence so that curve passes through these data points makes an authentic surface. The final data points are obtained by doing such a specific sorting.

B. Surface parameterization

Surface Parameterization is the determination of all relevant surface data such as knot vectors, control points. In this paper, B-spline surface is used for fitting the data points. A B-spline surface is defined in terms of a (m+1) × (n+1) grid of control points, and a set of B-Spline basis functions. The points on the B-spline surface [13], [14], [19] at a particular parametric value (u, v) are defined as:

$$P(u,v) = \sum_{i=0}^{n} \sum_{j=0}^{m} P_{ij} N_{i,k}(u)\, N_{j,l}(v) \tag{1}$$

Where P_{ij} is a control point, and $N_{i,k}$ and $N_{j,l}$ are basis functions in bi-parametric u and v directions, respectively. The ith B-Spline basis function $N_{i,k}(u)$ of order k (equivalent to degree k-1) is defined by the recurrence relations

$$N_{i,1}(u) = \begin{cases} 1 & \text{if } u_i \le u \le u_{i+1} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

$$N_{i,k}(u) = \left(\frac{u - u_i}{u_{i+k-1} - u_i}\right) N_{i,k-1}(u) + \left(\frac{u_{i+k} - u}{u_{i+k} - u_{i+1}}\right) N_{i+1,k-1}(u) \tag{3}$$

Let u = {u_0, u_1, …, u_n} be a non decreasing sequence of real numbers called knots and u is called knot vector. The number of times a knot appears in the knot vector is called the multiplicity of the knot and has an important effect on the shape and properties of the associated basis functions. Knot vectors can be classified into two groups. The first one is the uniform knot vector in which each knot appears only once and the distance between consecutive knots is always the same. A qualitatively different behaviour is obtained when any of the knots appears more than once, this case is referred as non-uniform knot vector. The most common case of non-uniform knot vectors consists of repeating the end knots as many times as the order while interior knots appear only once. Such a knot vector is called non-periodic knot vector [19], [27].

The values of u depend on whether the B-spline curve is an open (non-periodic) or closed (periodic) curve [19], [27]. For an open curve, it is given as

$$u_j = \begin{cases} 0, & j < k \\ j - k + 1, & k \le j \le n \\ n - k + 2, & j > n \end{cases} \quad (4)$$

Where $0 \le j \le n + k$ and the range of u is $0 \le u \le n - k + 2$.

For a closed B-spline curves the basis function [19], [27] is given as

$$N_{i,k}(u) = N_{0,k}((u - i + n + 1)\mathrm{mod}(n + 1)) \quad (5)$$

$$u_j = j, \ 0 \le j \le n + 1 \quad (6)$$

Where $0 < i \le n + 1$ and the range of u is $0 \le u \le n + 1$.

C. Control points calculation

The control points are first determined by analytical method using chord length approximation method [11], [16], [19]. The data points are D = [x, y, z] and from Eq. (1) in matrix form

$$[D] = [C][B] \quad (7)$$

Where $C_{i,j} = N_{i,k} N_{j,l}$. [D] is an (R+1) * (S+1) × 3 matrix containing the 3D coordinate of surface data points, [C] is an (R+1)*(S+1) × (n+1)*(m+1) matrix of the products of the B-spline basis function, and [B] is an (n+1)*(m+1) × 3 matrix of the 3D coordinate of the required polygon net points. If [C] is square, the defining polygon net is obtained by $[B]=[C]^{-1}[D]$ and if [C] is not square then $[B]=\left[[C]^T[C]\right]^{-1}[C]^T[D]$. For R data points the parameter value at the lth data points in the u parametric direction is obtained as

$$u_1 = 0, \qquad \frac{u_l}{u_{max}} = \frac{\sum_{g=2}^{l} \left|D_{g,s} - D_{g-1,s}\right|}{\sum_{g=2}^{r} \left|D_{g,s} - D_{g-1,s}\right|} \quad (8)$$

Similarly, for S data points in the w parametric direction is obtained.

The second method for computing control points is least square method with genetic algorithm approach. This function is minimized by using genetic algorithm optimization approach. The cost function [2], [11] is approximated as

$$\mathrm{Min}\ Z = f(P) = \sum_{r=0}^{R} \sum_{s=0}^{S} \left| \sum_{i=0}^{n} \sum_{j=0}^{m} N_{i,k}(u_r)\, N_{j,l}(v_s)\, P_{i,j} - Q_{r,s} \right|^2 \quad (9)$$

A genetic algorithm (GA) has been recognized as one of the most powerful computational technique for optimization and global search problems. It is originated from the seminal work of John Holland in the 70s; they combine bio-inspired processes emulating genetic evolution namely natural selection, mutation and crossover in order to describe the growth and development of populations associated with the objective problem [11], [26]. In this paper, Genetic algorithm (GA) is used because it strongly differs in conception from other search methods, including traditional optimization methods and other stochastic search methods. The basic difference is that while other methods always process single points in the search space, genetic algorithms maintain a population of potential solutions [18]. In this method the balance between preserving feasible solution and rejecting infeasible ones is easy to get. A binary representation is needed to describe each individual in the population of interest. Each individual is made up of a sequence of binary bits (0 and 1). The string length and population size denote the length of the binary sequence and the number of individuals involved in the population [6].

In simple GA three genetic operators reproduction, crossover, mutation are used to formed a new population string [7], [8], [18], [20]. The selected individuals are called parents and the resulting individuals are called offspring. The new populations are obtained using a selection process (reproduction) based on individual adaptation. The individuals with the best adaptation measure have more chance of reproducing and generating new individuals by crossing and muting. The reproduction operator can be implemented in several ways, such as tournament, proportionate, roulette wheel, rank-based, hall of fame, Boltzmann selection, etc. In this paper rank based selection method is used.

In rank based selection individual's rank is used to calculate the selection probability instead of the fitness function. The last individual gives a better chance of chromosomes with small fitness values to take part in the reproduction. The individuals are arranged in ascending order according to the value of their fitness functions. Afterwards, each chromosome obtains a rank depending on the place it has taken in this order [7], [20], [21]. Thus, the worst chromosome, which is first in the order, has a rank 0, while for the best one, which is the last—the rank is $(\mu-1)$, where μ is population size. GA comprises two types of ranking selection, linear and square, here linear ranking is used [20]. According to linear ranking the selection probability is proportional to the rank of each individual i and calculates as follows

$$\Pr(i) = \frac{\alpha + \left[\frac{\text{rank}(i)}{\mu - 1}\right](\beta - \alpha)}{\mu} \quad (10)$$

Where β is selective pressure represents the expected number of offspring to be allocated to the best individual, while α refers to the worst one and β must range in the interval $1 \le \beta \le 2$. Here in linear ranking $\beta = 2$ is taken as default and the parameter α calculated as $\alpha = 2 - \beta$ [20].

After reproduction, a crossover operator with a crossover probability creates two new individuals (offspring) by combining parts from two randomly selected individuals of the population. In GA the crossover operator is randomly applied with the specific probability known as crossover probability. Normally, the probability for crossover ranges from 0.6 to 0.95 [6]. A good GA performance requires the choice of a high crossover probability. If no crossover was performed, offspring is the exact copy of parents. The various crossover techniques are single point crossover, two point crossover, multi-point crossover (n-point crossover), uniform crossover, three parent crossover, shuffle crossover, ordered crossover etc. In this paper, uniform crossover is used. According to it, each gene of each child creates by randomly selecting respective gene from one of both parents. Both parents have an equal chance to contribute in the children's chromosomes [18], [20], [21].

Mutation is a unitary transformation which creates, with a mutation probability, a new individual by making modifications to one selected individual. A good algorithm performance requires the choice of a low mutation probability. Mutation of a bit involves flipping a bit, changing 0 to 1 and vice-versa. Mutation is done by flipping, interchanging, and reversing. Mutation is needed to keep diversity in the population. Normally, the probability for bit mutation ranges from 0.001 to 0.01 [6], [20], [21].

The new population obtained after mutation completes one generation of GA. This population is used to find fitness value, if termination criterion is not satisfied then it goes to reproduction process and this iterative process continues until the stopping criterion is satisfied. After optimization, the control points [22] is obtained as

$$\text{vec}(Q) = M \cdot \text{vec}(P)$$

$$M^T \cdot \text{vec}(Q) = M^T \cdot M \cdot \text{vec}(P) \quad (11)$$

Equation (11) can be solved by using SVD (singular value decomposition) [1], [23].The matrix M is decomposed as $M=U*S*V^T$. Here M^T*M is a square matrix, and its inverse is obtained as

$$[M^T \cdot M]^{-1} = V.\left[\text{diag}\left(\frac{1}{\emptyset_k}\right)\right] \cdot U^T \quad (12)$$

Where U = a column orthogonal matrix, S = a diagonal matrix with the elements $\emptyset_k$ called as singular values, V = a square orthogonal matrix. Then from equation (11) and (12) vec(P) [16],[17] is obtained

$$[M^T \cdot M]^{-1} \cdot M^T \cdot \text{vec}(Q) = \text{vec}(P) \quad (13)$$

After this, all output of previous steps is collected in order to perform surface reconstruction. Reconstructed data points are then compared to original data points according to some error measure such as the root mean squared error.

III. RESULT

An example of a known object shape (cylinder having radius 5 and height 5 unit) is B-spline surface parameterized to evaluate the control point placement based on genetic algorithm. The cylinders are to be one of the most basic geometric shapes. Since the cylinder is a closed surface in one parametric directions but open in the other one, so the choice of knot vectors is not trivial. Thus, a cylinder is a good to test the performance of method against strictly rational geometric shapes. The proposed method applied on a set of 3876 number 3D data points. The surface has been modelled for seven control points and it is shown in Fig. 2(b) and its top view with the data points and the control points obtained by genetic algorithm approach is shown in Fig. 2(a). The values of GA parameter used in optimization are presented in Table I. The root mean square (RMS) error between the modelled curve and data points for different number of control points (cp) is reported as in Table II. The curve now conforms better to the data points. The fitness is increases (the error decreases) while the generations are increasing.

TABLE I: THE SET OF PARAMETER OF GA OPTIMIZATION RUN

Parameter	GA Values
Population size	10
String length	10×2×Number of control point (for 5cp, String length=100)
Crossover rate	0.5 (Uniform Crossover)
Mutation rate	0.01
Generation	500

TABLE II: THE RMS VALUES OF MODELLED CURVE AND DATA POINTS FOR GA

Generation	Best RMS for 5cp	Best RMS for 7cp
Initial	0.4225	0.0790
10	0.2799	0.0372
25	0.2575	0.0277
50	0.1922	0.0210
100	0.1881	0.0181
200	0.1747	0.0179
300	0.1726	0.0187
400	0.1779	0.0184
500	0.1715	0.0174

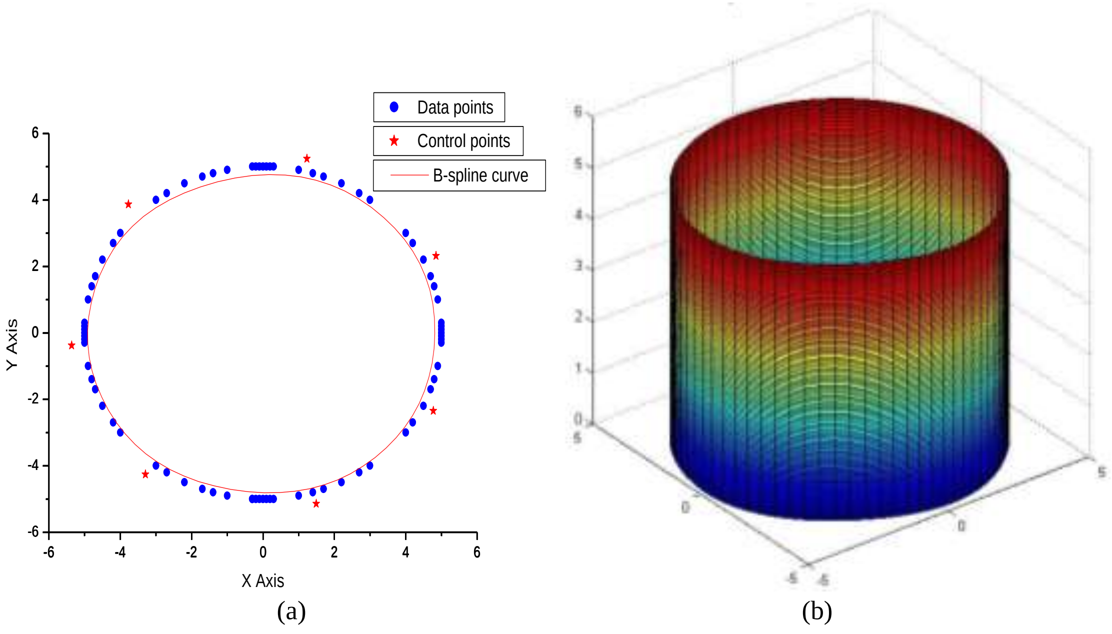

Fig. 2. (a) Upper view of a right circular cylinder fitting surface with data points and control points obtained by GA approach (b) Reconstructed surface of cylinder by closed B-spline curve for 7 control points

The fitting surface has been obtained for (2, 2) order surface with a net of 2×7 control points, with the least error of 0.0164 by genetic algorithm approach and 0.0174 by analytical method. The error is of the order of 10^{-3} in the data points, although other values for the order and number of control points also produced good errors. The order (2, 2) means that the surface is linear in the vertical direction (actually a ruled surface), while it is a circle comprised of quadratic pieces in the horizontal direction. Fig. 2(a) displays the surface as seen from the top, so that we can see the good fitting to data point along the most complicated direction (in fact, shape in vertical direction is linear and hence trivial) and Fig. 2(b) displays the reconstructed surface. The fitness value changes for different number of control points. The variation of fitness according to variation in number of control points is reported in Table III. It shows that fitness is best (the least error) between 25 – 30 number of control points and its variation is shown in Fig. 5.

TABLE III: FITNESS VARIATIONS WITH NUMBER OF CONTROL POINTS

Number of Control Points	Best Fitness	Generation
5	0.1640	255
6	0.0413	325
7	0.0164	297
8	0.0083	485
9	0.0063	206
10	0.0047	249
15	0.00226	219
20	0.00113	146
25	0.00107	316
30	0.00123	310
35	0.00166	182
40	0.00187	401

IV. CONCLUSION

In this paper, the introduced method is an efficient GA based B-spline surface reconstruction from clouds of 3D data points. The method relies on the GA approach to obtain the relevant parameters in order to construct the B-spline surface that fits the data points better. The feature of this method is the proper parameterization of data points and the very accurate fitting of the surface to the clouds of points. The results show that the proposed method enables the proper selection of the control points so that reconstructed surface will have minimum error and reduced computational effort. Genetic Algorithms have global perspective and robust. However, their convergence is slower and computation cost is more for generations which are much closer to optimal solution. The variation of fitness with GA generation is shown in Fig. 3.

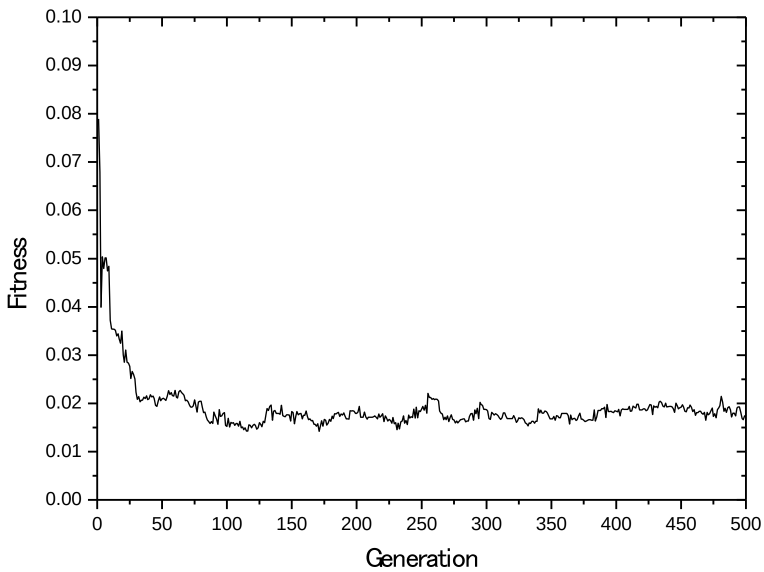

Fig. 3 GA based parameter optimization according to generations for 7 control points

The comparative analysis between analytical method and genetic algorithm based approach to obtain the position of control points is shown in Fig. 4. The least error obtained by analytical method is 0.0174 where as the least error obtained by genetic algorithm based approach is 0.0164 for 7 number of control points. This shows 5.7471% improvement in least error by genetic algorithm based approach. The use of genetic algorithm based approach gives an efficient improvement to obtain control point. The proposed method also gives efficient improvement for other number of control points. The variation of number of control points with the fitness is as shown in Fig. 5. The analytical method gives least error 0.001132 for 28 number of control points where as the genetic algorithm based approach gives the least error for number of control points 25 – 30 with the efficient improvement in the least error. Thus the genetic algorithm based approach of obtaining control points is an effective approach with the optimum number of points and proper placement of control points with the least value of error.

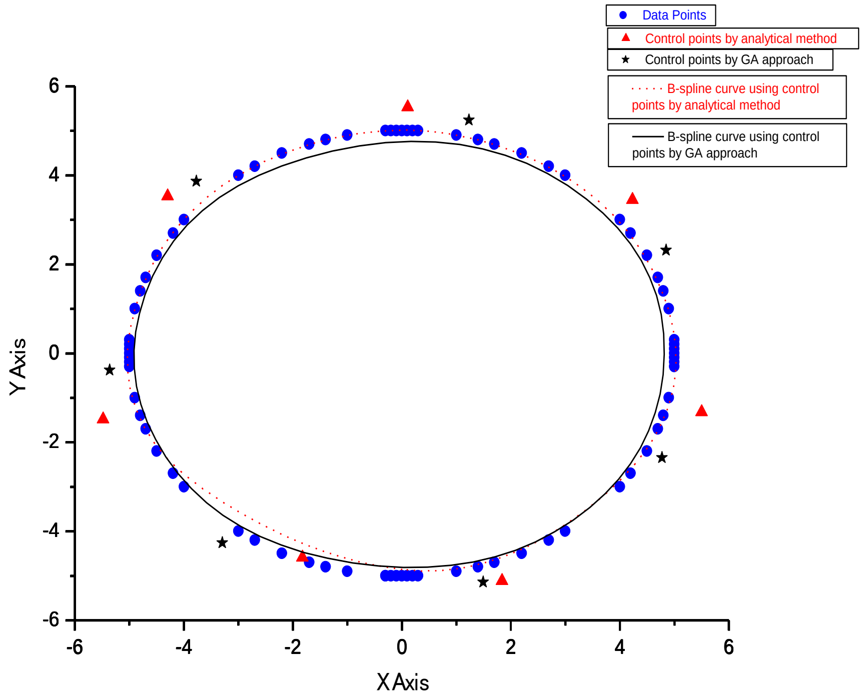

Fig. 4 Comparative analysis between analytical method and genetic algorithm based approach

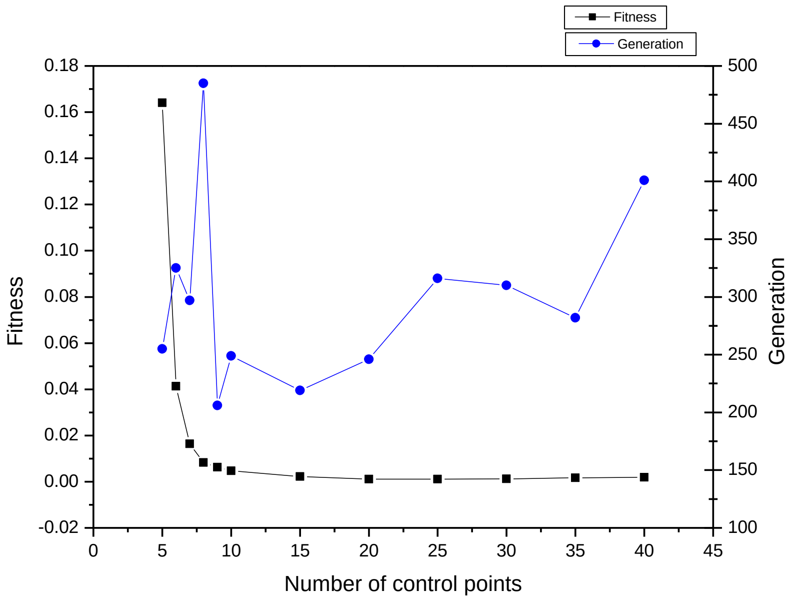

Fig. 5 Fitness variations with number of control points with the respective GA generation

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